# Transmission Line Positive Sequence Parameter Estimation Using Synchronized Measurements at Both Ends 

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## SUMMARY

In bulk power system planning and operations, several studies are performed to facilitate development and ensure reliable, secure, and economical operation. These power system studies typically include steady-state and transient studies, which are used to assess the impact of various proposed power system developments and credible disturbances imposed on the bulk power system that is initially operated at nominal frequency and known secure state. However, to obtain credible results from these studies, it is important to employ effective and efficient study algorithms and develop reasonably accurate models that adequately represent the behaviour of the bulk power system elements under steady-state and transient conditions. Three-phase overhead transmission lines typically constitute the largest proportion of the bulk power system elements, so it's important to give careful attention to the accuracy and robustness of the method(s) used to determine their model parameters.

Except for three-phase overhead transmission lines, the model parameters of all other power system elements can be determined relatively easily in isolation. The sequence model parameters of threephase overhead transmission lines are typically determined using classical geometric methods such as the Carson's method and other derivatives. However, it has been shown that errors may be introduced in the model parameters of three-phase overhead transmission lines when classical geometric methods are used. This is mainly due to the simplifying assumptions that are normally used for practical overhead transmission lines [1].

With the introduction of modern computers, intelligent electronic metering such as phasor measurement units (PMUs), global positioning systems (GPS), and remote communication technology, it is now possible to accurately determine the positive sequence parameters of in-service three-phase overhead transmission lines without knowing their geometric arrangement or other physical construction details. The proposed method presented in this paper uses current technology to determine the positive sequence model parameters of in-service three-phase overhead transmission lines and is represented using the Lumped Pi-Circuit model. In developing the proposed method, bus frequency, substation bus voltage vector, and real and reactive power flow injections at both ends of a three-phase overhead transmission line, are synchronously monitored using PMUs. The synchronized measurements obtained from the PMUs at both ends of the three-phase overhead transmission line are transmitted to a control centre where the time-stamped measurement data is presented in real-time and archived in a database, if required.

The positive sequence series resistance, series reactance and charging susceptance of the three-phase overhead transmission line's Lumped Pi-Circuit model are expressed in terms of the bus voltage vector, real and reactive power flow injections at both ends of the three-phase overhead transmission line. Ideally, the synchronized measurement data is sorted by bus terminal frequency to extract the data records with nominal frequency. The positive sequence Lumped Pi-Circuit model parameters of the three-phase overhead transmission line are then determined by taking the average over several samples.
The proposed method described in this paper may be used to corroborate the positive sequence parameters obtained from the classical geometric methods. This is particularly useful when simplifying assumptions must be made in cases where the three-phase overhead transmission lines pass over irregular terrain with wide variations in soil resistivity and geometric conductor arrangements. The proposed method may also be useful to assess the real-time variations in transmission line model parameters for use in the dynamic MVA rating of three-phase overhead transmission lines.

The proposed method is useful only when the three-phase overhead transmission line is in service, and GPS-synchronized measurements can be obtained from both ends. In such a scenario only the positive and negative sequence parameters of the transmission line can be determined.
The accuracy of the proposed method was verified using ten (10) concurrent samples of the positive sequence voltage vector, and real and reactive power flow injections at both ends of a selected transmission line in a sample PSS/e power flow case. To mimic the variations in practical in-service overhead transmission lines, many load/generation scenarios were established to vary the power flow in the selected transmission line in the PSS/e power flow case. The positive sequence Lumped PiCircuit model parameters of the transmission line, as determined by the proposed method, are shown to be reasonably consistent with that modelled in the sample PSS/e power flow case.

## KEYWORDS

Three-phase, overhead transmission lines, geometric method, Lumped Pi-Circuit Model, PMU, positive sequence, resistance, reactance, susceptance, voltage vector, power flow.

## POSITIVE SEQUENCE TRANSMISSION LINE PARAMETERS IN STUDIES

In bulk power system planning and operations, several studies are performed to facilitate power system developments and operate the power system in a reliable, secure, and economical manner. These studies are normally steady-state and transient. Typically, steady-state studies include shortcircuit and power flow where post-disturbance transient effects are ignored, and the power system has had sufficient time to reach new steady-state conditions. Transient studies include stability and electromagnetic where the power system frequency may deviate from its nominal value for relatively short periods. To perform steady-state and transient studies, it is necessary to develop appropriate models for all the power system elements so that these models account for the static and dynamic nature of the condition under investigation and the impact on the bulk power system. As three-phase overhead transmission lines typically constitute the largest proportion of the bulk power system elements, it's important that careful attention be given to the accuracy and robustness of the method(s) used to determine their model parameters.

Mathematical models are used to represent the steady-state and transient characteristics of the bulk power system elements such as transformers, three-phase overhead transmission lines and underground/submarine transmission cables, conventional and renewable generation, capacitors, reactors, and static and dynamic power compensating devices. Except for three-phase overhead transmission lines, the model parameters of all other power system elements are relatively easy to determine in isolation. Whereas in the classical methods, the physical, geometric, and routing/terrain information specific to the three-phase overhead transmission line is required to determine its model parameters.
The positive sequence parameters of a three-phase overhead transmission line are particularly useful in steady-state balanced off-line power flow studies. However, significant errors in the positive sequence model parameters of three-phase overhead transmission lines may result in the improper distribution of real and reactive power flows and bus voltage vectors throughout the modelled power system. It has also been shown that errors in transmission line parameters can significantly affect the accuracy of the state estimator and other downstream online applications [2]-[4]. Hence there is value in benchmarking the positive sequence parameters of three-phase overhead transmission lines as determined by the classical geometric methods versus non-geometric measurement-based methods.

## SEQUENCE IMPEDANCE MATRIX OF TRANSMISSION LINES

To determine the model parameters of three-phase overhead transmission lines using classical geometric methods, knowledge of the conductor type, geometric arrangement of the conductors, and average earth resistivity over the right-of-way of the three-phase overhead transmission line are required. Carson's and other related geometric methods are commonly used methods to calculate the positive, negative, and zero sequence impedances of three-phase overhead transmission lines. However, Carson's theoretical three-phase overhead transmission line has no sag and runs parallel to a flat homogeneous earth. Whereas practical three-phase overhead transmission lines are more likely to sag between supporting towers, change geometric arrangement as tower type changes, and be routed over multi-elevation terrain with non-homogenous soil type.

To improve the accuracy of the geometric-based methods, finite element analysis has been employed to break up the transmission line into small elements so that each increment of the transmission line closely approximates to Carson's theoretical transmission line. This requires a 3-D CAD representation of the proposed or as-built transmission line, and knowledge of the conductor type and earth resistivity along the transmission line's right-of-way. The execution speed of the finite element software depends on the selected mesh size, which is normally a compromise to minimize the execution time of the finite element software program. However, using the finite element method for evaluating the sequence model parameters of three-phase overhead transmission lines has been shown to give reasonably acceptable results [1].

The sequence series impedance $\left(Z_{\text {series }}^{012}\right)$ parameter and shunt charging susceptance $\left(Y_{\text {shunt }}^{012}\right)$ matrices of a three-phase overhead transmission line is denoted by the following expressions. The off-diagonal terms of the matrices are usually insignificant for perfectly transposed transmission lines.
$Z_{\text {series }}^{012}=\left[\begin{array}{ccc}Z_{00} & 0 & 0 \\ 0 & Z_{11} & 0 \\ 0 & 0 & Z_{22}\end{array}\right]$

Where:
$Z_{00} \rightarrow$ Zero sequence series impedance of transmission line
$Z_{11} \rightarrow$ Positive sequence series impedance of transmission line
$Z_{22} \rightarrow$ Negative sequence series impedance of transmission line
$Y_{\text {shunt }}^{012}=\left[\begin{array}{ccc}Y_{00} & 0 & 0 \\ 0 & Y_{11} & 0 \\ 0 & 0 & Y_{22}\end{array}\right]$

Where:
$Y_{00} \rightarrow$ Zero sequence shunt charging susceptance of transmission line
$Y_{11} \rightarrow$ Positive sequence shunt charging susceptance of transmission line
$Y_{22} \rightarrow$ Negative sequence shunt charging susceptance of transmission line

## PMU AND DATA ACQUISITION SYSTEM

Figure 1 illustrates the typical setup of the potential and current transformers used for data acquisition of bus voltages and current injections, respectively, at either ends of a transmission line [5]. Only a single phase of the three-phase system is shown. The PMU samples and monitors the post-processed bus voltage and current injection from the secondary outputs of voltage and current transformers, respectively. The PMU then processes the three-phase bus voltages and current injections as vectors relative to an established zero-degree angle reference based on its interface to a GPS antenna. The positive, negative, and zero sequence voltage and current injection samples are evaluated using an established transformation matrix, and results are stored in the PMU memory and/or transmitted to a local data collector such as a remote terminal unit (RTU).

Typically, the time-synchronized voltage, current injection vectors, and frequency data are conveyed to a central control centre via a microwave transmission medium. The SCADA system at the control centre processes the voltage and current vectors, presents real-time values to the operators, and archives said data, if required. The SCADA access to the voltage and current injection vectors at both ends of the transmission lines provides the opportunity to calculate the estimates of the transmission line parameters at regular intervals [5]. The power flow quantities that are useful for the method as proposed in this paper are the positive sequence real and reactive power injections and bus voltages at each transmission line terminal. Hence, the positive sequence sending and receiving terminals' real and reactive power are functions of the bus voltage, $\tilde{V}_{s}, \widetilde{V}_{r}$ and current injection, $\tilde{I}_{s}, \tilde{I}_{r}$ vectors and are evaluated as follows:
$P_{s}=\operatorname{real}\left(\widetilde{V}_{s} \tilde{I}_{s}^{*}\right)$
$Q_{s}=\operatorname{imag}\left(\tilde{V}_{s} \tilde{I}_{s}^{*}\right)$
$P_{r}=\operatorname{real}\left(\tilde{V}_{r} \tilde{I}_{r}^{*}\right)$
$Q_{r}=\operatorname{imag}\left(\widetilde{V}_{r} \tilde{I}_{r}^{*}\right)$


Figure 1: Time-Synchronized Voltage and Current Vector Acquisition at one Terminal of Transmission Line ${ }^{1}$

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## PROPOSED TRANSMISSION LINE PARAMETER ESTIMATION METHOD

Figure 2 illustrates the Lumped Pi-Circuit model of a three-phase overhead transmission line used in the proposed method to represent the positive sequence model of the transmission line. PMUs monitor the positive sequence voltage vectors and real and reactive power injections at both transmission line terminals.


Figure 2: Positive Sequence Lumped Pi-Circuit Model of Three-Phase Overhead Transmission Line
Where:
$P_{s} \rightarrow$ Sending end real power injection (pu)
$P_{r} \rightarrow$ Receiving end real power injection (pu)
$Q_{s} \rightarrow$ Sending end reactive power injection (pu)
$Q_{r} \rightarrow$ Receiving end reactive power injection (pu)
$V_{s} \rightarrow$ Sending end voltage magnitude (pu)
$V_{r} \rightarrow$ Receiving end voltage magnitude (pu)
$\theta_{s r} \rightarrow$ Voltage angle difference across transmission line (rad)
$\theta_{s r}=\theta_{s}-\theta_{r}$

The total positive sequence per unit charging shunt admittance $\left(y_{s h}\right)$ of the transmission line is evaluated using the following expressions [7]:

$$
y_{s h}=g_{s h}+j b_{s h}
$$

Where:

$$
\begin{gather*}
g_{s h}=\frac{2\left[V_{r}\left(P_{s} \cos \theta_{s r}+Q_{s} \sin \theta_{s r}\right)+P_{r} V_{s}\right]\left(V_{s} \cos \theta_{s r}+V_{r}\right)}{V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}+V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]}+  \tag{1}\\
\frac{2\left[V_{r}\left(P_{s} \sin \theta_{s r}-Q_{s} \cos \theta_{s r}\right)-Q_{r} V_{s}\right] V_{s} \sin \theta_{s r}}{V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}+V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]} \\
b_{s h}=\frac{-2\left[V_{r}\left(P_{s} \cos \theta_{s r}+Q_{s} \sin \theta_{s r}\right)+P_{r} V_{s}\right] V_{s} \sin \theta_{s r}}{V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}+V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]}+  \tag{2}\\
\frac{2\left[V_{r}\left(P_{s} \sin \theta_{s r}-Q_{s} \cos \theta_{s r}\right)-Q_{r} V_{s}\right]\left(V_{s} \cos \theta_{s r}+V_{r}\right)}{V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}+V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]}
\end{gather*}
$$

The positive sequence per unit series admittance $\left(y_{s}\right)$ of the transmission line is evaluated using the following expressions [7]:
$y_{s}=g_{s}+j b_{s}$
Where:
$g_{s}=\frac{\left[V_{r}\left(P_{s} \cos \theta_{s r}+Q_{s} \sin \theta_{s r}\right)-P_{r} V_{s}\right]\left(V_{s} \cos \theta_{s r}-V_{r}\right)}{2 V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}-V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]}+$
$\frac{\left[V_{r}\left(P_{s} \sin \theta_{s r}-Q_{s} \cos \theta_{s r}\right)+Q_{r} V_{s}\right] V_{s} \sin \theta_{s r}}{2 V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}-V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]}$
$b_{s}=\frac{-\left[V_{r}\left(P_{s} \cos \theta_{s r}+Q_{s} \sin \theta_{s r}\right)-P_{r} V_{s}\right] V_{s} \sin \theta_{s r}}{2 V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}-V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]}+$
$\left[V_{r}\left(P_{s} \sin \theta_{s r}-Q_{s} \cos \theta_{s r}\right)+Q_{r} V_{s}\right]\left(V_{s} \cos \theta_{s r}-V_{r}\right)$
$2 V_{s} V_{r}\left[\left(V_{s} \cos \theta_{s r}-V_{r}\right)^{2}+\left(V_{s} \sin \theta_{s r}\right)^{2}\right]$
So, the positive sequence series impedance $\left(z_{s}\right)$ of the transmission line is evaluated using the following expression:
$z_{s}=r_{s}+j x_{s}=\frac{g_{s}-j b_{s}}{g_{s}{ }^{2}+b_{s}{ }^{2}}$

## VALIDATION OF PROPOSED METHOD

To validate the proposed method, power flow data on a 138 kV transmission line in a sample PSS/e case with positive sequence series resistance of $\mathbf{0 . 0 8} \mathrm{pu}$, series reactance of $\mathbf{j 0 . 2 4} \mathrm{pu}$ and total charging susceptance of $\mathbf{j 0 . 0 5}$ pu was used. The model parameters and real and reactive power flows are on a 100 MVA base. Ten (10) unique power flow scenarios were developed to provide 10 records of positive sequence voltage vector and real and reactive power at both terminals of the selected line as shown in Table 1.

Table 1: Samples of Positive Sequence Transmission Line Power Flow Results

| Sample | $V_{s}$ <br> $(p u)$ | $P_{s}$ <br> $(p u)$ | $Q_{s}$ <br> $(p u)$ | $V_{r}$ <br> $(p u)$ | $P_{r}$ <br> $(p u)$ | $Q_{r}$ <br> $(p u)$ | $\theta_{s r}$ <br> $(\mathrm{rad})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.050 | 0.4362 | 0.0959 | 0.9927 | -0.4213 | -0.1033 | 0.0911 |
| 2 | 1.050 | 0.2023 | 0.0780 | 1.0112 | -0.1985 | -0.1198 | 0.0378 |
| 3 | 1.050 | 0.3026 | 0.1282 | 0.9930 | -0.2942 | -0.1552 | 0.0577 |
| 4 | 1.050 | 0.5630 | 0.1016 | 0.9848 | -0.5388 | -0.0808 | 0.1210 |
| 5 | 1.050 | 0.5086 | 0.0811 | 0.9923 | -0.4889 | -0.0744 | 0.1090 |
| 6 | 1.050 | 0.3894 | 0.1071 | 0.9927 | -0.3771 | -0.1224 | 0.0794 |
| 7 | 1.050 | 0.4362 | 0.0959 | 0.9927 | -0.4212 | -0.1034 | 0.0911 |
| 8 | 1.050 | 0.4380 | 0.0955 | 0.9927 | -0.4230 | -0.1026 | 0.0915 |
| 9 | 1.050 | 0.2413 | 0.1453 | 0.9927 | -0.2349 | -0.1784 | 0.0423 |
| 10 | 1.050 | 0.4309 | 0.0970 | 0.9930 | -0.4163 | -0.1054 | 0.0898 |

Expressions (1) through (5), detailed in the previous section, were programmed in MATLAB and used to evaluate the transmission line model parameters corresponding to the power flow results shown in Table 1. The average values of the series resistance, series reactance and total charging susceptance shown in Table 2 are reasonably consistent with that used in the sample PSS/e power flow case.

Table 2: Transmission Line Positive Sequence Parameters Calculated at Each Sample

| Sample | $g_{s}$ <br> $(p u)$ | $b_{s}$ <br> $(p u)$ | $r_{s}$ <br> $(p u)$ | $x_{s}$ <br> $(p u)$ | $g_{s h}$ <br> $(p u)$ | $b_{s h}$ <br> $(p u)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.2531 | -3.7375 | $\mathbf{0 . 0 8 0 6 4}$ | $\mathbf{0 . 2 4 0 5}$ | - | $\mathbf{0 . 0 5 0}$ |
| 2 | 1.2413 | -3.7375 | $\mathbf{0 . 0 8 0 0 3}$ | $\mathbf{0 . 2 4 1 0}$ | - | $\mathbf{0 . 0 5 0}$ |
| 3 | 1.2549 | -3.7375 | $\mathbf{0 . 0 8 0 7 3}$ | $\mathbf{0 . 2 4 0 5}$ | - | $\mathbf{0 . 0 5 0}$ |
| 4 | 1.2541 | -3.7375 | $\mathbf{0 . 0 8 0 6 9}$ | $\mathbf{0 . 2 4 0 5}$ | - | $\mathbf{0 . 0 5 0}$ |
| 5 | 1.2521 | -3.7375 | $\mathbf{0 . 0 8 0 5 9}$ | $\mathbf{0 . 2 4 0 6}$ | - | $\mathbf{0 . 0 5 0}$ |
| 6 | 1.2538 | -3.7375 | $\mathbf{0 . 0 8 0 6 8}$ | $\mathbf{0 . 2 4 0 5}$ | - | $\mathbf{0 . 0 5 0}$ |
| 7 | 1.2531 | -3.7375 | $\mathbf{0 . 0 8 0 6 4}$ | $\mathbf{0 . 2 4 0 5}$ | - | $\mathbf{0 . 0 5 0}$ |
| 8 | 1.2530 | -3.7375 | $\mathbf{0 . 0 8 0 6 4}$ | $\mathbf{0 . 2 4 0 5}$ | - | $\mathbf{0 . 0 5 0}$ |
| 9 | 1.2555 | -3.7375 | $\mathbf{0 . 0 8 0 7 6}$ | $\mathbf{0 . 2 4 0 4}$ | - | $\mathbf{0 . 0 5 0}$ |
| 10 | 1.2531 | -3.7375 | $\mathbf{0 . 0 8 0 6 4}$ | $\mathbf{0 . 2 4 0 5}$ | - | $\mathbf{0 . 0 5 0}$ |

## DISCUSSION AND CONCULSION

This paper proposed a non-geometric measurement-based method used to estimate the positive sequence model parameters of a three-phase overhead transmission line. The proposed method employs synchronized measurements of certain power flow quantities at both terminals of in-service three-phase overhead transmission lines. The model parameters, as estimated by the proposed method, may vary slightly due to variations in transmission line conductor temperature, skin-effect, and changes in frequency. However, provided that the source data from the current and voltage transformers are reasonably accurate and the appropriate conversion multipliers are used, the average values (at nominal frequency) over a certain number of samples would provide reasonable estimates of the real-time transmission line model parameters.

Currently, simplifying assumptions are used in the classical geometric methods to evaluate the sequence parameters of overhead transmission lines. These simplifying assumptions may introduce errors that could impact the results in steady-state and transient studies. However, these errors may be mitigated using finite element solution methods, which have been shown to give reasonably good results.

At a minimum, the proposed non-geometric measurement-based method provides a benchmark for assessing the accuracy of the classical geometric methods and may be useful to inspire a high degree of confidence in the transmission line model parameters used in bulk power system online and off-line studies. The proposed method may also be useful to assess the real-time variations in transmission line model parameters for use in the dynamic MVA rating of three-phase overhead transmission lines.

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